

On Large N Solution of Gaiotto-Tomasiello Theory

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Abstract

The planar solution is discussed for an $\mathcal{N} = 3$ Chern-Simons-matter theory constructed recently by Gaiotto and Tomasiello. The planar resolvent is obtained in terms of contour integrals. If the sum of two Chern-Simons levels k_1, k_2 is small, the expectation value of a supersymmetric Wilson loop grows exponentially with the total 't Hooft coupling, as is expected from AdS/CFT correspondence. If one of the Chern-Simons levels, say k_2 , is taken to infinity, for which one of the 't Hooft coupling constants becomes zero, then the exponential behavior disappears.

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1 Introduction

Wilson loop operators have played important roles in the study of AdS/CFT correspondence [1]. It was proposed in [2][3][4] that the expectation value $\langle W \rangle$ of a Wilson loop operator in a gauge theory would be equal to the partition function of a fundamental string in a certain string theory. In the most well-studied example of AdS/CFT correspondence, the gauge theory is $\mathcal{N} = 4$ super Yang-Mills theory, and the string theory is Type IIB string theory on $\text{AdS}_5 \times S^5$. A consequence of the proposal is that, in the limit in which the string coupling constant is small and the radius of the AdS space is large in the string side, $\langle W \rangle$ is evaluated in terms of the area of a minimal surface in the AdS_5 space. Since the limit corresponds to the large 't Hooft coupling limit in the gauge theory, the result so obtained should be quite non-trivial.

A similar relation between Wilson loops and strings was already proposed long time ago. Recently, there was surprising progress on this relation in the context of AdS/CFT correspondence. In the case where the gauge theory is $\mathcal{N} = 4$ super Yang-Mills theory, a quantitative check of the proposal [2][3][4] was performed perturbatively in [5], and then the proposal was proved in the above-mentioned limit using the localization technique in [6]. For another case where the gauge theory is ABJM theory [7], a similar localization technique for various Chern-Simons-matter theories was developed in [8] which can apply to supersymmetric Wilson loops constructed in [9][10][11][12]. The localization formula obtained in [8] was then studied in the planar limit [13][14], and it was confirmed that the proposal was indeed valid for ABJM theory [14]. In addition to this result, the planar solution of ABJM theory was further investigated in [15] and the mysterious $N^{\frac{3}{2}}$ behavior of the free energy, which is another consequence of AdS/CFT correspondence, was derived without relying on calculations based on string theory or M-theory.

It would be interesting if the investigations mentioned above could be extended to more general superconformal gauge theories. The large 't Hooft coupling λ behavior of Wilson loops was investigated in [16] for the planar limit of $\mathcal{N} = 2$ quiver gauge theory of \hat{A}_1 -type and $\mathcal{N} = 2$ supersymmetric gauge theory with $N_f = 2N_c$ fundamental matters. The arguments in [16] were based on the localization of [6]. In fact, it is quite difficult to analyze the large λ limit even in the planar limit. Although the localization allows us to reduce the problems to the corresponding problems in a matrix model, the matrix model is still complicated for $\mathcal{N} = 2$ theories due to the complicated structure of the 1-loop determinant around the fixed point for the localization. Note that the above two theories are related. The former theory has the gauge group $\text{SU}(N) \times \text{SU}(N)$. Let g_1, g_2 be the gauge coupling constants for those gauge fields. The latter theory can be obtained (up to a decoupled system) from the former theory by taking one of the gauge coupling constant, say g_2 , to zero. See also [17]. This relation between two theories should descend to a relation for Wilson loops.

In [16], it was argued that a suitable Wilson loop in the quiver gauge theory with $g_1 = g_2$ would behave as

$$\langle W \rangle \sim e^{\sqrt{2\lambda}} \quad (1.1)$$

in the large λ limit, as expected from AdS/CFT correspondence. In the following, we will describe that a Wilson loop has the AdS behavior when the dependence of $\langle W \rangle$ on λ in the large λ limit coincides with the one expected from AdS/CFT correspondence. In this terminology, it was argued that a suitable linear combination of Wilson loops in the quiver gauge theory has the AdS behavior. On the other hand,

the Wilson loop in the gauge theory with fundamental matters does not seem to have the AdS behavior. This would be rather natural from the point of view of the field theory: the Wilson loop would be simply screened by the fundamental matters. Note that the contribution of the fundamental matters is not negligible in the planar limit which keeps the conformal symmetry, since the number of the fundamental matters should also become large in the limit, or one has to take the Veneziano limit, not the 't Hooft limit. However, the anticipated non-exponential behavior does not seem to be easily understood from the possible gravity dual point of view. Since the number of fundamental matters is large, it is not appropriate to realize them in terms of probe D-branes in a similar manner as in [18][19][20][21][58][59][60]. It was argued in [16] that a possible resolution might be related to the contributions from many, possibly infinite, saddle-points for the string partition function, if they existed. All of them were supposed to be of the same order of magnitude but might have different phases so that the leading AdS behavior could be canceled among them, leaving sub-leading contributions which could be non-exponential.

Since this observation looks quite interesting, and there could be new insights in this kind of situations, it would be worth performing similar investigation in more details. Due to the complexity of the above example, it would be better to find a simpler setup which may share the same physics. The theory to be studied should have a product gauge group, and there should exist a limit in which some of the gauge fields decouple. If the theory before taking the limit has bi-fundamental matters, then there are a large number of fundamental matters after taking the limit. It would be interesting if $\langle W \rangle$ could be evaluated in such a theory.

In this paper, we investigate a Chern-Simons-matter theory constructed recently in [22]. The theory has a product gauge group with two adjustable coupling constants. By taking one of the coupling constant to zero, it is reduced to another Chern-Simons-matter theory which contains a large number of fundamental matters. The localization calculation of [8] can apply to both the original theory and the limit theory. We find that the original theory has a range of the coupling constants in which a Wilson loop has the AdS behavior, while the corresponding Wilson loop in the limit theory behaves at most as a power of the 't Hooft coupling constant. Since the localization formula for Chern-Simons-matter theory is much simpler than that for superconformal gauge theories in four dimensions, the results mentioned above can be obtained *exactly* in the planar limit. It should be noted that the investigation of the Chern-Simons-matter theory might not be a good analogy with the system considered in [16] since the gauge fields are non-dynamical in the former.

This paper is organized as follows. In section 2, we review the construction of [22] and some properties of their gravity duals. Section 3 contains the analysis of the planar solution of an $\mathcal{N} = 3$ theory in [22]. The details of the theory is summarized in subsection 3.1, and the planar resolvent of the theory is determined in subsection 3.2. The technique used here is a standard one found, for example, in [45]. The resolvent is given in terms of two contour integrals, supplemented by two equations which determine two parameters in the integrals in terms of the coupling constants of the theory. Subsection 3.3 shows the planar analysis for the limit theory. Section 4 is devoted to discussion. In Appendix A, we review basic facts on $\mathcal{N} = 3$ Chern-Simons-matter theories. For the check of our planar solution obtained in subsection 3.2, in Appendix B we derive the planar resolvent obtained in [52], which is related to ABJ theory via a suitable analytic continuation [14], from our solution. A comment on the consistency of the

$\mathcal{N} = 3$ theory is made in Appendix C.

2 Chern-Simons levels and Romans mass

As explained in the introduction, we would like to find a Chern-Simons-matter theory which has a product gauge group, and in which one can take a limit such that the gauge fields for one of the simple factors of the gauge group decouple. One may think that ABJ theory [23] would be the right setup. Indeed, in ABJ theory, there are two 't Hooft coupling constants

$$\lambda_1 = \frac{N_1}{k}, \quad \lambda_2 = \frac{N_2}{k}. \quad (2.1)$$

It is possible to take a limit in which all N_1, N_2, k are large while λ_1, λ_2 are kept finite, and then take $\lambda_2 \rightarrow 0$ limit. Such a limit was recently investigated in [15]. However, this limit implies $N_1 \gg N_2$. As a result, the number N_f of the fundamental matters, which come from bi-fundamental matters in ABJ theory via the decoupling, turns out to be small compared with the rank N_1 of the surviving gauge group $U(N_1)$ since N_f is proportional to N_2 . In this case, the effect of the fundamental matters is just a small perturbation, and their effect on the geometry in a possible gravity dual would be irrelevant.

Therefore, we would like to find another theory. It turns out that a theory constructed in [22] is suitable for our purpose. The theory has the gauge group $U(N_1) \times U(N_2)$, the same group as in ABJ theory². The Chern-Simons levels k_1, k_2 for those $U(N)$ factors, respectively, do not need to satisfy $k_1 + k_2 = 0$. The relevant coupling constants of this theory are

$$\lambda_1 = \frac{N_1}{k_1}, \quad \lambda_2 = \frac{N_2}{k_2}. \quad (2.2)$$

In the limit $k_2 \rightarrow \infty$, the gauge fields for $U(N_2)$ would decouple, and a bi-fundamental matter becomes N_2 fundamental matters of $U(N_1)$. Since N_2 can be taken to be of the order of N_1 , the contribution of the fundamental matters remains relevant even after taking the planar limit. The price to pay is that the theories in [22] have at most $\mathcal{N} = 3$ supersymmetry. Actually, this is not a bad deal. Indeed, $\mathcal{N} = 3$ supersymmetry is powerful enough to write down the Lagrangian of the IR fixed point theory explicitly, and more importantly, it makes the localization calculation of [8] available. The planar solution of GT theory will be discussed in the next section. In the remainder of this section, we will review some properties of the possible gravity duals of those theories argued in [22].

The crucial difference of the theories in [22] from ABJ theory is that, as mentioned above, $k_1 + k_2$ is non-zero. It was argued in [22] that $k_1 + k_2$ is related to the R-R zero-form field strength F_0 , or the Romans mass, in the dual Type IIA string theory. In other words, the gravity duals of the theories in [22] should be massive Type IIA string theories.

² Chern-Simons-matter theories with different levels were also discussed in [61].

A piece of evidence for this claim is as follows. Consider a D0-brane. If a non-zero F_0 is turned on, then the worldvolume gauge fields A_μ on the D0-brane have the coupling of the form [24][25]

$$\int_{\text{D0}} F_0 A. \quad (2.3)$$

This means that there is a tadpole for A_μ . To cancel this tadpole, an appropriate number of fundamental strings should be attached to the D0-branes. In fact, the number turns out to be F_0 , so that F_0 must be quantized. If the background is of an AdS-type, then the opposite ends of those fundamental strings should be attached to the boundary of the AdS space. AdS/CFT correspondence claims that the dual boundary field theory should have a local operator corresponding to the D0-brane with F_0 fundamental strings. Due to the presence of the endpoints of the fundamental strings on the boundary, the local operator should be in a non-trivial representation R of a gauge group. For example, if the boundary field theory would have the gauge group $\text{SU}(N)$, then R should be an irreducible component of a product of F_0 fundamental representations of $\text{SU}(N)$.

There is indeed such an operator in a Chern-Simons-matter theory if it has the gauge group $\text{U}(N_1) \times \text{U}(N_2)$ with the levels k_1, k_2 , respectively, such that

$$k_1 + k_2 = F_0. \quad (2.4)$$

In gauge theories, there exists a so-called monopole operator [26][27][28] which creates a Dirac monopole-like gauge field configuration on \mathbb{R}^3 . Let the monopole be at the origin O of \mathbb{R}^3 . It is necessary to introduce a covering of $\mathbb{R}^3 \setminus \{O\}$ to construct a non-trivial gauge bundle for the monopole. The charge of the monopole is encoded in the transition functions of the bundle. One may use a singular gauge transformation to trivialize the transition functions, leaving a Dirac string connecting O and the point at infinity. If the gauge theory has a Chern-Simons term, then it was shown in [29][30] that the singular gauge transformation also provides an insertion of a Wilson line along the Dirac string³. In the simplest case where the gauge group is $\text{U}(1)$ and the monopole has the unit magnetic charge, the resulting Wilson line is the one describing the propagation of a particle with the electric charge k .

In the theories discussed in [22], one can construct a so-called di-monopole operator which is a product of two monopole operators for two $\text{U}(N)$ factors. Let us first consider the case $N_1 = N_2 = 1$ for simplicity. By the same argument, it can be shown that the di-monopole operator is equivalent to a Wilson line corresponding to a charged particle with the electric charges (k_1, k_2) . It would be natural to relate the electric charges to endpoints of fundamental strings. If the levels are chosen such that $k_1, -k_2 > 0$, then the di-monopole operator should correspond to a state with k_1 strings with an orientation and $|k_2|$ strings with the opposite orientation. A pair of string endpoints with opposite orientations can be contracted with themselves or with bi-fundamental fields, so the di-monopole operator would be in a representation which is obtained, for example, from a product of $k_1 + k_2$ fundamental representation of $\text{SU}(N_1)$ when $k_1 + k_2$ is positive. Therefore, under the assumption (2.4), one may expect that the di-monopole operator is the dual of the D0-brane with F_0 fundamental strings attached. Since the ground state of a fundamental string stretched between a D2-brane and a D0-brane is bosonic in the trivial flat

³ More precisely, one starts with cutting out a semi-infinite tube in \mathbb{R}^3 which contains the Dirac string. Since this procedure creates a boundary, a suitable boundary action should be added to the Chern-Simons action. Then, the gauge transformation (which is now regular due to the elimination of the tube) provides a boundary term which is equivalent to a Wilson line inserted in the path-integral.

background, it would be natural to expect that this is the case even in the presence of F_0 , at least when F_0 is small. This suggests that the representation of the Wilson line corresponding to the di-monopole operator is $(\text{Sym}\mathbf{N}_1^{k_1}, \text{Sym}\bar{\mathbf{N}}_2^{k_2})$ for general N_1 and N_2 . Although the Wilson line operator looks non-local, the di-monopole operator is a local operator since the Dirac string is unphysical. Monopole operators play important roles in Chern-Simons-matter theories [31][32][33][34][35][36][37][57][62].

The relation between the Chern-Simons levels and F_0 was further elucidated in [38]. In [38], a brane configuration for the $\mathcal{N} = 0$ version of the theory in [22] was proposed in Type IIB string theory. It is basically the same brane configuration used in [7], and a non-zero Romans mass is introduced by letting D7-branes pass through the system. In a T-dual Type IIA string theory, this process corresponds to introducing D8-branes which pass through the system. Since D7-branes create a cut for R-R zero form field C_0 , the effect of the D7-branes still remains even when they are far apart. The cut for C_0 contributes to the Chern-Simons levels, and as a result, the levels which were originally $(k, -k)$ is turned into $(k, -k+1)$ when a single D7-brane passes through. Repeating this process, or using anti-D7-branes, one can change k_2 to an arbitrary value without changing k_1 .

The explicit gravity solutions were obtained in [39][22] which were proposed to be the dual gravity backgrounds for the $\mathcal{N} = 0, 1$ theories in [22]. Recent researches on solutions of massive Type IIA supergravity are found in [40][41][42][43].

3 Planar analysis on Gaiotto-Tomasiello theory

3.1 Gaiotto-Tomasiello theory

In this section, we consider the planar limit of a Chern-Simons-matter theory constructed in [22]. The theory has $\mathcal{N} = 3$ supersymmetry, and the gauge group is $U(N_1) \times U(N_2)$ with the Chern-Simons levels k_1 and k_2 , respectively. It is not necessary to set $k_1 + k_2 = 0$. The gauge fields couple to bi-fundamental matter fields. The action is

$$S_{\text{GT}} = S_{\text{CS}} + \int d^3x \left[\int d^4\theta \left(\text{tr}_2(A_i^\dagger e^{-2V^{(1)}} A_i e^{2V^{(2)}}) + \text{tr}_1(B_i^\dagger e^{-2V^{(2)}} B_i e^{2V^{(1)}}) \right) \right] \\ + \int d^3x \left[\int d^2\theta \left(\frac{2\pi}{k_1} \text{tr}_1(A_i B_i A_j B_j) + \frac{2\pi}{k_2} \text{tr}_2(B_i A_i B_j A_j) \right) + \text{h.c.} \right] \quad (3.1)$$

Here A_i ($i = 1, 2$) are $\mathcal{N} = 2$ chiral superfields in the bi-fundamental representation of $U(N_1) \times U(N_2)$, B_i are $\mathcal{N} = 2$ chiral superfields in the anti-bi-fundamental representation. $V^{(s)}$ ($s = 1, 2$) are $\mathcal{N} = 2$ vector superfields for $U(N_s)$. The traces tr_s are the ones for fundamental representations of $U(N_s)$. The part S_{CS} in terms of component fields is

$$S_{\text{CS}} = \sum_{s=1}^2 \frac{k_s}{4\pi} \int d^3x \text{tr}_s \left[\epsilon^{\mu\nu\rho} \left(A_\mu^{(s)} \partial_\nu A_\rho^{(s)} - \frac{2i}{3} A_\mu^{(s)} A_\nu^{(s)} A_\rho^{(s)} \right) - 2i \bar{\lambda}^{(s)} \lambda^{(s)} - 2D^{(s)} \sigma^{(s)} \right]. \quad (3.2)$$

This theory is known to be conformal even quantum mechanically, irrespective of the values of the parameters. In the following, we will refer to this theory as Gaiotto-Tomasiello (GT) theory. Some

properties of $\mathcal{N} = 3$ Chern-Simons-matter theories are summarized in Appendix A. See also [44].

Similarly to ABJ theory, GT theory may not be defined for all possible values of the parameters. By an argument similar to the one in [23], it may be suggested that GT theory with parameters satisfying

$$N_1 - N_2 > k_1 > 0 \quad (3.3)$$

would be inconsistent as a quantum field theory. See Appendix C for more details. In the following, we consider GT theory with parameters which does not satisfy (3.3), assuming that the theory would be consistent. Note that one can vary k_2 without spoiling the assumed consistency.

Since GT theory is superconformal with enough number of supercharges, the localization calculation of [8] can apply to GT theory as well when it is defined on S^3 . It was shown in [8] that the resulting expression for the partition function is independent of the details of the superpotential. Since GT theory has exactly the same gauge group and the matter content with ABJ theory, the localization formula for the partition function can be obtained from that of ABJ theory by simply changing the Chern-Simons levels from $(k, -k)$ to (k_1, k_2) . The explicit expression is

$$Z_{\text{GT}} = \int \prod_{i=1}^{N_1} d\phi_i \prod_{a=1}^{N_2} d\tilde{\phi}_a e^{-S(\phi, \tilde{\phi})}, \quad (3.4)$$

where

$$e^{-S(\phi, \tilde{\phi})} = e^{i\pi \left(\sum_i k_1 \phi_i^2 + k_2 \sum_a \tilde{\phi}_a^2 \right)} \frac{\prod_{i < j} \sinh^2[\pi(\phi_i - \phi_j)] \prod_{a < b} \sinh^2[\pi(\tilde{\phi}_a - \tilde{\phi}_b)]}{\prod_{i,a} \cosh^2[\pi(\phi_i - \tilde{\phi}_a)]}. \quad (3.5)$$

Supersymmetric Wilson loops can be considered in Chern-Simons-matter theories with at least $\mathcal{N} = 2$ supersymmetry [44]. The explicit form for GT theory is

$$W_s[C] = \frac{1}{N_s} \text{tr}_s \text{P exp} \left[\oint_C d\tau (i\dot{x}^\mu A_\mu^{(s)} + |\dot{x}| \sigma^{(s)}) \right], \quad (3.6)$$

where the contour C is a great circle on S^3 , and $x^\mu = x^\mu(\tau)$ is a parametrization of C . These operators preserve half of (manifest) $\mathcal{N} = 2$ supersymmetry.

Note that the Wilson loops (3.6) can be also considered in ABJM theory. These are actually equivalent to 1/6-BPS Wilson loops constructed in [9][10][11] and studied further in [13][14][15]. For example, $W_1[C]$ corresponds in ABJM theory to

$$W[C] = \frac{1}{N_1} \text{tr}_1 \text{P exp} \left[\oint_C d\tau (i\dot{x}^\mu A_\mu^{(1)} + \frac{2\pi}{k} |\dot{x}| M_I^J Y^I Y_J) \right], \quad (3.7)$$

where $M_I^J = \text{diag}(+1, +1, -1, -1)$ and $Y^I = (A_i, B_i^\dagger)$. This correspondence can be checked as follows. The action of ABJM theory can be written in terms of $\mathcal{N} = 2$ superfields [7]. The auxiliary field $\sigma^{(1)}$ can be integrated out by the constraint

$$\sigma^{(1)} = \frac{2\pi}{k} (A_i A_i^\dagger - B_i^\dagger B_i) \quad (3.8)$$

obtained by integrating out another auxiliary field $D^{(1)}$. Inserting this into (3.6), then one obtains the 1/6-BPS Wilson loop (3.7).

The field $\sigma^{(1)}$ is a component of a triplet of $SU(2)_R$ R-symmetry in the $\mathcal{N} = 3$ theory. There is a family of supersymmetric Wilson loops which are related to (3.7) by R-symmetry rotations. Explicitly,

$$W_1[C, \theta] = \frac{1}{N_1} \text{tr}_1 \text{P exp} \left[\oint_C d\tau (i\dot{x}^\mu A_\mu^{(1)} + |\dot{x}| \theta_i \varphi^i) \right], \quad (3.9)$$

where φ^i ($i = 1, 2, 3$) consist of $\sigma^{(1)}$ and an adjoint chiral scalar field⁴, and θ_i is a unit three-vector. This is actually a special case of the operator constructed in [44]. Since φ^i are auxiliary fields, they can be integrated out. Then (3.7) is obtained again with

$$M_I^J = [\theta_i \sigma^i \otimes \mathbf{1}_2]_I^J, \quad (3.10)$$

where $\mathbf{1}_2$ is the identity 2×2 matrix.

In a UV completion, for example the worldvolume theory on a brane configuration in [7], the fields φ^i are dynamical, describing motions of the D-branes in transverse directions. In this UV point of view, (3.9) looks very much like the 1/2-BPS Wilson loop in $\mathcal{N} = 4$ super Yang-Mills theory in four dimensions. This seems to suggest that the information of the vector θ_i which specifies the direction along which a dual string is stretched would be encoded in the matrix M_I^J in (3.7).

In GT theory, one can consider both (3.6) and (3.9). Since (3.9) is obtained from (3.6) by an R-symmetry rotation, let us focus on (3.6). The expectation value of the Wilson loop (3.6) is expressed in terms of a finite-dimensional integral

$$\langle W_1[C] \rangle = \frac{1}{Z_{\text{GT}}} \int \prod_{i=1}^{N_1} d\phi_i \prod_{a=1}^{N_2} d\tilde{\phi}_a e^{-S(\phi, \tilde{\phi})} \frac{1}{N_1} \sum_{i=1}^{N_1} e^{2\pi\phi_i} \quad (3.11)$$

using the localization [8].

3.2 Planar limit

We would like to concentrate on the planar limit

$$k_1, k_2, N_1, N_2 \propto k, \quad k \rightarrow \infty. \quad (3.12)$$

In this limit, every observable which can be written in terms of ϕ_i and $\tilde{\phi}_a$ is determined by the solution of the saddle-point equations derived from $S(\phi, \tilde{\phi})$ in (3.5). In terms of rescaled variables

$$u_i := 2\pi\phi_i, \quad v_a := 2\pi\tilde{\phi}_a, \quad (3.13)$$

the saddle-point equations are

$$-\frac{ik_1}{2\pi} u_i = \sum_{j \neq i}^{N_1} \coth \frac{u_i - u_j}{2} - \sum_{a=1}^{N_2} \tanh \frac{u_i - v_a}{2}, \quad (3.14)$$

$$-\frac{ik_2}{2\pi} v_a = \sum_{b \neq a}^{N_2} \coth \frac{v_a - v_b}{2} - \sum_{i=1}^{N_1} \tanh \frac{v_a - u_i}{2}. \quad (3.15)$$

⁴ The adjoint chiral scalar fields are already integrated out in (3.1). See Appendix A.

As in [14], we consider instead the following equations

$$\frac{k_1}{2\pi} u_i = \sum_{j \neq i}^{N_1} \coth \frac{u_i - u_j}{2} + \sum_{a=1}^{N_2} \tanh \frac{u_i - v_a}{2}, \quad (3.16)$$

$$\frac{k_2}{2\pi} v_a = \sum_{b \neq a}^{N_2} \coth \frac{v_a - v_b}{2} + \sum_{i=1}^{N_1} \tanh \frac{v_a - u_i}{2}, \quad (3.17)$$

where k_1, k_2 are assumed to be positive, and the solution of the original equations (3.14)(3.15) will be obtained via a suitable analytic continuation.

Dividing by k , one obtains

$$\kappa_1 u_i = \frac{t_1}{N_1} \sum_{j \neq i}^{N_1} \coth \frac{u_i - u_j}{2} + \frac{t_2}{N_2} \sum_{a=1}^{N_2} \tanh \frac{u_i - v_a}{2}, \quad (3.18)$$

$$\kappa_2 v_a = \frac{t_2}{N_2} \sum_{b \neq a}^{N_2} \coth \frac{v_a - v_b}{2} + \frac{t_1}{N_1} \sum_{i=1}^{N_1} \tanh \frac{v_a - u_i}{2}, \quad (3.19)$$

where

$$\kappa_{1,2} := \frac{k_{1,2}}{k}, \quad t_{1,2} := \frac{2\pi N_{1,2}}{k}. \quad (3.20)$$

The saddle-point equations for ABJ theory correspond to $\kappa_{1,2} = 1$.

Define new variables

$$z_i := e^{u_i}, \quad w_a := e^{v_a}. \quad (3.21)$$

In terms of these variables, the equations (3.18)(3.19) are written as

$$\kappa_1 \log z_i = t_1 \frac{N_1 - 1}{N_1} + t_2 + \frac{2t_1}{N_1} \sum_{j \neq i}^{N_1} \frac{z_j}{z_i - z_j} - \frac{2t_2}{N_2} \sum_{a=1}^{N_2} \frac{w_a}{z_i + w_a}, \quad (3.22)$$

$$\kappa_2 \log w_a = t_1 + t_2 \frac{N_2 - 1}{N_2} + \frac{2t_2}{N_2} \sum_{b \neq a}^{N_2} \frac{w_b}{w_a - w_b} - \frac{2t_1}{N_1} \sum_{i=1}^{N_1} \frac{z_i}{w_a + z_i}. \quad (3.23)$$

Let us assume that all z_i and all w_a are real. Define the following resolvent

$$v(z) := t_1 \int dx \rho(x) \frac{x}{z - x} - t_2 \int dx \tilde{\rho}(x) \frac{x}{z + x}, \quad (3.24)$$

where $\rho(x)$ and $\tilde{\rho}(x)$ are defined formally as

$$\rho(x) := \frac{1}{N_1} \sum_{i=1}^{N_1} \delta(x - z_i), \quad \tilde{\rho}(x) := \frac{1}{N_2} \sum_{a=1}^{N_2} \delta(x - w_a). \quad (3.25)$$

Note that the overall scale of $v(z)$ depends on the choice of k and therefore irrelevant.

Assuming that both z_i and w_a form continuous segments on the real axis in the planar limit (3.12), $v(z)$ has two cuts on the z -plane. Let $[a, b], [c, d] \subset \mathbb{R}$ be the cuts of $v(z)$ corresponding to $-w_a$ and z_i , respectively. In terms of $v(z)$, the equations (3.22)(3.23) are written as

$$\kappa_1 \log y - t = v(y + i0) + v(y - i0), \quad (c < y < d) \quad (3.26)$$

$$\kappa_2 \log(-y) - t = v(y + i0) + v(y - i0), \quad (a < y < b) \quad (3.27)$$

where $t = t_1 + t_2$. These equations suggest $b < 0 < c$.

It will turn out that the function

$$F(z, s) := \sqrt{(z-a)(z-b)(z-c)(z-d)} \int_c^d dx \frac{f_+(x, s)}{z-x}, \quad (3.28)$$

where s is a real-valued parameter and

$$f_{\pm}(x, s) := \frac{\pm \log(\pm e^{-s}x)}{\sqrt{|(x-a)(x-b)(x-c)(x-d)|}}, \quad (3.29)$$

is a building block of the solution. The phase of the square-root in $F(z, s)$ is chosen such that

$$\sqrt{(z-a)(z-b)(z-c)(z-d)} = +z^2 + O(z) \quad (3.30)$$

for $z \rightarrow +\infty$.

Choose y such that $c < y < d$. One can show that

$$F(y \pm i0, s) = \pi \log(e^{-s}y) \pm i \sqrt{|(y-a)(y-b)(y-c)(y-d)|} \int_c^d dx \mathcal{P} \frac{f_+(x, s)}{y-x}. \quad (3.31)$$

Therefore, $F(z, s)$ satisfies

$$F(y + i0, s) + F(y - i0, s) = 2\pi \log(e^{-s}y). \quad (c < y < d) \quad (3.32)$$

Next, choose y such that $a < y < b$. Then

$$F(y \pm i0, s) = \mp i \sqrt{|(y-a)(y-b)(y-c)(y-d)|} \int_c^d dx \mathcal{P} \frac{f_+(x, s)}{y-x}, \quad (3.33)$$

and therefore,

$$F(y + i0, s) + F(y - i0, s) = 0. \quad (a < y < b) \quad (3.34)$$

Similarly, the other function defined as

$$G(z, s) := \sqrt{(z-a)(z-b)(z-c)(z-d)} \int_a^b dx \frac{f_-(x, s)}{z-x} \quad (3.35)$$

satisfies

$$G(y + i0, s) + G(y - i0, s) = 0, \quad (c < y < d) \quad (3.36)$$

$$G(y + i0, s) + G(y - i0, s) = 2\pi \log(-e^{-s}y). \quad (a < y < b) \quad (3.37)$$

It is now easy to check that

$$v(z) = \frac{\kappa_1}{2\pi} F\left(z, \frac{t}{\kappa_1}\right) + \frac{\kappa_2}{2\pi} G\left(z, \frac{t}{\kappa_2}\right) \quad (3.38)$$

is the solution of the saddle-point equations (3.26)(3.27). Note that a change in k only changes the overall scale, as it should be.

The parameters a, b, c, d are constrained by requiring a suitable behavior of $v(z)$ expected from the definition (3.24). It is obvious that $v(z)$ must satisfy

$$v(z) = \begin{cases} O(z^{-1}), & (z \rightarrow \infty) \\ -t. & (z = 0) \end{cases} \quad (3.39)$$

These conditions imply

$$\frac{\kappa_1}{2\pi} \int_c^d dx f_+ \left(x, \frac{t}{\kappa_1} \right) + \frac{\kappa_2}{2\pi} \int_a^b dx f_- \left(x, \frac{t}{\kappa_2} \right) = 0, \quad (3.40)$$

$$\begin{aligned} & \frac{\kappa_1}{2\pi} \int_c^d dx \left(x - \frac{a+b+c+d}{2} \right) f_+ \left(x, \frac{t}{\kappa_1} \right) \\ & + \frac{\kappa_2}{2\pi} \int_a^b dx \left(x - \frac{a+b+c+d}{2} \right) f_- \left(x, \frac{t}{\kappa_2} \right) = 0, \end{aligned} \quad (3.41)$$

$$\sqrt{abcd} \left[\frac{\kappa_1}{2\pi} \int_c^d dx x^{-1} f_+ \left(x, \frac{t}{\kappa_1} \right) + \frac{\kappa_2}{2\pi} \int_a^b dx x^{-1} f_- \left(x, \frac{t}{\kappa_2} \right) \right] = -t. \quad (3.42)$$

These three conditions are not enough to determine four parameters. In fact, this is a situation analogous to the one found in multi-cut solutions of one-matrix models (See, for example, [45]). In the context of the one-matrix model, one can impose another condition which forbids the tunneling of the eigenvalues from one cut to another. In the present context, this kind of condition is not available since the eigenvalues z_i are distinct from w_a .

Noticing that the above conditions are expressed in terms of only t , not including t_1 nor t_2 separately, one of the followings

$$\oint_{C_+} \frac{dz}{2\pi i} \frac{v(z)}{z} = t_1, \quad \oint_{C_-} \frac{dz}{2\pi i} \frac{v(z)}{z} = t_2, \quad (3.43)$$

which are derived from the definition (3.24), is an independent condition. Here C_+ is a contour encircles the cut $[c, d]$, and C_- encircles $[a, b]$.

It is possible to simplify the problem. Notice that the original saddle-point equations (3.16)(3.17) are invariant under the simultaneous sign flip of u_i and v_a . This fact implies that the eigenvalue distributions of z_i and w_a should be invariant under the inversion $z \rightarrow z^{-1}$ and $w \rightarrow w^{-1}$. We would like to make an ansatz

$$ab = 1, \quad cd = 1. \quad (3.44)$$

However, this ansatz already fixes two parameters in terms of the others, so it should be checked whether the above conditions (3.40)(3.41)(3.42) are compatible with this ansatz. Indeed, this is the case. Using (3.40), the second condition (3.41) can be written as

$$\frac{\kappa_1}{2\pi} \int_c^d dx x f_+ \left(x, \frac{t}{\kappa_1} \right) + \frac{\kappa_2}{2\pi} \int_a^b dx x f_- \left(x, \frac{t}{\kappa_2} \right) = 0. \quad (3.45)$$

Changing the variable to $y = x^{-1}$, this can be written as

$$\begin{aligned} -\frac{\kappa_1}{2\pi} \int_c^d dy y^{-1} f_+ \left(y, \frac{t}{\kappa_1} \right) - \frac{\kappa_2}{2\pi} \int_a^b dy y^{-1} f_- \left(y, \frac{t}{\kappa_2} \right) &= \frac{t}{\pi} \int_c^d \frac{dy}{y \sqrt{|(y-a)(y-b)(y-c)(y-d)|}} \\ &\quad - \frac{t}{\pi} \int_a^b \frac{dy}{y \sqrt{|(y-a)(y-b)(y-c)(y-d)|}} \\ &= t. \end{aligned} \quad (3.46)$$

This coincides with (3.42) under the ansatz (3.44).

Moreover, the first condition (3.40) is trivially satisfied under the ansatz (3.44). By the change of the variable $x \rightarrow x^{-1}$, one can check that the identities

$$\int_c^d dx f_+(x, 0) = 0 = \int_a^b dx f_-(x, 0) \quad (3.47)$$

hold. Then, the condition (3.40) becomes

$$\int_c^d \frac{dx}{\sqrt{|(x-a)(x-b)(x-c)(x-d)|}} - \int_a^b \frac{dx}{\sqrt{|(x-a)(x-b)(x-c)(x-d)|}} = 0. \quad (3.48)$$

It is easy to show that this is an identity.

Therefore, it is found that there is in fact a single condition for two parameters, verifying the ansatz (3.44). Still another condition is necessary to determine the parameters completely. One can choose it from one of (3.43). Equivalently, one can choose both of (3.43) as the conditions.

In summary, we obtained the planar resolvent

$$v(z) = \sqrt{(z-a)(z-b)(z-c)(z-d)} \left[\frac{\kappa_1}{2\pi} \int_c^d dx \frac{f_+(x, \frac{t}{\kappa_1})}{z-x} + \frac{\kappa_2}{2\pi} \int_a^b dx \frac{f_-(x, \frac{t}{\kappa_2})}{z-x} \right], \quad (3.49)$$

where the parameters are determined by the conditions

$$ab = 1, \quad cd = 1, \quad \oint_{C_+} \frac{dz}{2\pi i} \frac{v(z)}{z} = t_1, \quad \oint_{C_-} \frac{dz}{2\pi i} \frac{v(z)}{z} = t_2. \quad (3.50)$$

Recall that we introduced an auxiliary parameter k in order that the dependence of the resolvent on the parameters becomes symmetric. The above equations imply that the parameters a, b, c, d are independent of the choice of k , as it should be the case. On the other hand, $v(z)$ is proportional to k^{-1} due to the definition (3.24).

It is rather straightforward to show that the solution (3.49) is reduced to the ABJ solution in [14] when $\kappa_1 = \kappa_2$, or $k_1 = k_2 = k$. Appendix B contains the details of the calculations.

It was shown in [14] that the expectation value $\langle W_1[C] \rangle$ of the supersymmetric Wilson loop (3.6) behaves as

$$\langle W_1[C] \rangle \sim e^{\pi\sqrt{2\lambda}} \quad (3.51)$$

for large λ in ABJM theory, namely, in the case where $k_1 = -k_2 = k$, $N_1 = N_2 = N$, and $\lambda = \frac{N}{k}$. Therefore, the Wilson loop has the AdS behavior, confirming the conjecture of [7]. The behavior (3.51) must be preserved if the deviation of $k_1 + k_2$ from zero is negligible in the planar limit. This situation would correspond to a gravity solution, a perturbation of AdS₄ solution by a Romans mass, investigated in [46]. In the next subsection, we will consider the opposite limit in which $k_1 + k_2$ goes to infinity. In the limit, we will find that the AdS behavior (3.51) disappears.

3.3 Decoupling limit

It is interesting to consider the limit $|k_2| \rightarrow \infty$ in GT theory. If this limit is taken in the perturbation theory, it is obvious that the gauge fields for $U(N_2)$ decouple, and this $U(N_2)$ gauge symmetry becomes a global symmetry. As a result, a single (anti-)bi-fundamental field becomes N_2 (anti-)fundamental fields for $U(N_1)$ gauge group. This is simply because one of the 't Hooft coupling, $\frac{N_2}{k_2}$ goes to zero in the limit. This is true even non-perturbatively which can be checked by examining the localization formula (3.4). The limit $|k_2| \rightarrow \infty$ can be easily investigated in terms of the stationary phase method. Namely, all $\tilde{\phi}_a$ are forced to be zero in the limit, and the resulting partition function coincides with the one for $\mathcal{N} = 3$ Chern-Simons-matter theory with a single gauge group $U(N_1)$ coupled to N_2 fundamental matters and N_2 anti-fundamental matters⁵.

A similar limit can be considered in the planar limit discussed in the previous subsection. One can take

$$k_1, k_2, N_1, N_2 \propto k, \quad k \rightarrow +\infty, \quad \frac{k}{k_2} \rightarrow 0. \quad (3.52)$$

In this limit, the Gaussian terms in (3.5) are

$$i\pi \left[\frac{k_1}{N_1} \cdot N_1 \sum_{i=1}^{N_1} \phi_i^2 + \frac{k_2}{N_2} \cdot N_2 \sum_{a=1}^{N_2} \tilde{\phi}_a^2 \right]. \quad (3.53)$$

As long as N_1 and N_2 are the same order, and $\frac{N_1}{k_1}$ and $\frac{N_2}{k_2}$ are the same order, these two terms are comparable. In the limit

$$\frac{N_2}{k_2} = \frac{N_2}{k} \frac{k}{k_2} \rightarrow 0, \quad (3.54)$$

the second term dominates, and therefore $\tilde{\phi}_a$ are set to zero.

The planar solution of the limit theory may be investigated in terms of the planar resolvent (3.49) of GT theory. However, it is far easier to investigate directly the saddle-point equation of the limit theory. We would like to solve

$$u_i + t_2 \tanh \frac{u_i}{2} = \frac{t_1}{N_1} \sum_{j \neq i}^N \coth \frac{u_i - u_j}{2}, \quad (3.55)$$

where we choose $k = k_1$. The result of the limit theory will be obtained by continuing $k_1 \rightarrow -ik_1$ analytically.

Define the resolvent

$$v(z) := t_1 \int dx \rho(x) \frac{x}{z - x}. \quad (3.56)$$

In terms of $v(z)$, the equation (3.55) can be written as

$$\log(e^{-t_1} x) + t_2 \frac{x-1}{x+1} = v(x+i0) + v(x-i0). \quad (3.57)$$

⁵ More precisely, the partition function approaches that of the $\mathcal{N} = 3$ theory with (anti-)fundamental matters times a decoupled system of $\tilde{\phi}_a$ in the limit. The decoupled system can be neglected as long as one considers observables independent of $\tilde{\phi}_a$.

We assume that the solution of (3.57) has a single cut $[a, b]$ with $a > 0$. It is easy to check that

$$v(z) = v_0(z) + v_f(z) \quad (3.58)$$

satisfies the saddle-point equation (3.57), where

$$\begin{aligned} v_0(z) &= \frac{1}{\pi} \int_a^b dx \frac{1}{z-x} \frac{\sqrt{(z-a)(z-b)}}{\sqrt{|(x-a)(x-b)|}} \frac{\log(e^{-t_1}x)}{2} \\ &= \frac{1}{2} \log \left[\frac{e^{-t_1}}{2\sqrt{ab} + a + b} \left(z + \sqrt{ab} - \sqrt{(z-a)(z-b)} \right)^2 \right], \end{aligned} \quad (3.59)$$

$$\begin{aligned} v_f(z) &= \frac{1}{\pi} \int_a^b dx \frac{1}{z-x} \frac{\sqrt{(z-a)(z-b)}}{\sqrt{|(x-a)(x-b)|}} \frac{t_2}{2} \frac{x-1}{x+1} \\ &= \frac{t_2}{2} \frac{z-1}{z+1} - \frac{t_2}{\sqrt{(1+a)(1+b)}} \frac{\sqrt{(z-a)(z-b)}}{z+1}. \end{aligned} \quad (3.60)$$

The parameters a and b are determined by imposing

$$v(0) = -t_1, \quad v(\infty) = 0. \quad (3.61)$$

The condition $v(0) = -t_1$ implies

$$\frac{1}{2} \log \left[\frac{e^{-t_1} (2\sqrt{ab})^2}{2\sqrt{ab} + a + b} \right] - \frac{t_2}{2} + \frac{t_2 \sqrt{ab}}{\sqrt{(1+a)(1+b)}} = -t_1. \quad (3.62)$$

The other condition $v(\infty) = 0$ implies

$$\frac{1}{2} \log \left[\frac{e^{-t_1}}{2\sqrt{ab} + a + b} \left(\sqrt{ab} + \frac{a+b}{2} \right)^2 \right] + \frac{t_2}{2} - \frac{t_2}{\sqrt{(1+a)(1+b)}} = 0. \quad (3.63)$$

As in the case with a finite k_2 , these two equations are consistent with the ansatz $ab = 1$. Under this ansatz, the above two conditions become equivalent. The remaining equation to be solved is then

$$\log \frac{4}{2+a+a^{-1}} + t_2 \sqrt{\frac{4}{2+a+a^{-1}}} = t_2 - t_1. \quad (3.64)$$

For simplicity, let us focus on the case $t_1 = t_2$, that is, $N_1 = N_2$. The parameter a can be obtained by

$$a = e^{2u}, \quad 2 \cosh u \cdot \log \cosh u = t_1. \quad (3.65)$$

The expectation value of the Wilson loop is

$$\begin{aligned} \langle e^{2\pi\phi} \rangle &= \frac{1}{t_1} \lim_{z \rightarrow \infty} z v(z) \\ &= \frac{1}{t_1} \sinh^2 u + \cosh u - 1. \end{aligned} \quad (3.66)$$

Let us consider the limit $t_1 \rightarrow +i\infty$. Let $\cosh u = r e^{i\theta}$. Then

$$\cos \theta \cdot r \log r - \sin \theta \cdot r \theta = 0, \quad (3.67)$$

$$\sin \theta \cdot r \log r + \cos \theta \cdot r \theta = \frac{|t_1|}{2} \quad (3.68)$$

are satisfied. These imply

$$\log r = \theta \tan \theta, \quad \frac{r \log r}{\sin \theta} = \frac{|t_1|}{2}. \quad (3.69)$$

The limit $|t_1| \rightarrow \infty$ thus corresponds to $r \rightarrow \infty$ with $\theta \rightarrow \frac{\pi}{2} - 0$. Therefore,

$$|\cosh u| = r = o(t_1). \quad (3.70)$$

This then implies that

$$\langle e^{2\pi\phi} \rangle = o(t_1). \quad (3.71)$$

We found that the exponential behavior observed in the case of small $k_1 + k_2$ is modified to a behavior which is at most power-like. This is a similar phenomenon whose existence was suggested in [16] for $\mathcal{N} = 2$ superconformal gauge theories in four dimensions.

4 Discussion

We have considered the planar limit of Gaiotto-Tomasiello theory. The planar resolvent (3.49) was obtained in terms of contour integrals. The large 't Hooft coupling behavior of supersymmetric Wilson loops was discussed for GT theory and for a limit theory in which one of the gauge fields $A_\mu^{(2)}$ decoupled. We found that the Wilson loop has the AdS behavior if $k_1 + k_2$ is small, but the AdS behavior disappears in the limit $k_2 \rightarrow \infty$. This phenomenon is quite similar to the one observed in [16]. In this paper it was shown explicitly in the planar limit.

It does not seem to be easy to understand the dependence on k_2 of the behavior of the Wilson loop in the gravity dual point of view. One reason is that the gravity solutions of massive Type IIA theory are quite complicated. Another, possibly more crucial, reason is that the large k_2 limit corresponds to a limit in which a field strength F_0 becomes large. Since F_0 behaves as a cosmological constant, a large value of F_0 would make the curvature of the background geometry large, implying that a supergravity approximation would not be valid any more. One may argue that the non-exponential behavior found in subsection 3.3 would be due to the fact that the radius of the dual AdS_4 space might become small, and therefore, the minimal surface in AdS_4 would not be dominant. To justify this claim, one needs to find a reliable technique to analyze Type IIA string theory in such a background.

It would be necessary to elaborate on the planar solution in more details. Since our solution (3.49) is not written in terms of familiar functions, it looks difficult to extract information on the theory. However, although implicit, the planar resolvent is completely specified, so whether a necessary piece of information can be extracted from (3.49) is a technical issue. It would be very interesting if there would exist a variant of the technique used in [14] which is applicable to GT theory. Such a technique then may enable one to make it possible to show an interpolation of the Wilson loop from a finite k_2 to infinite k_2 . Although this limit can be taken rather straightforwardly at the level of the partition function (3.4), it would be plausible to analyze the limit entirely in terms of the planar resolvent (3.49). Note that there would not be any singular behavior as k_2 goes to infinity. Provided that GT theory with a chosen parameters N_1, N_2, k_1, k_2 is well-defined, the theory should be kept well-defined in the process of increasing k_2 , since

this is just a process of decreasing a coupling constant. One might wonder whether there would be a possibility that the discreteness of k_2 might change the conclusion. However, as far as the planar limit is concerned, there does not seem to be an indication that the discreteness plays a role for large k_2 (something different may happen for small k_2 which corresponds to a strong 't Hooft coupling). It would be also interesting to check the behavior of the free energy in the large 't Hooft coupling limit, as in [15], so as to find the number of the degrees of freedom of GT theory in the limit.

It would be interesting to extend our analysis to other Chern-Simons-matter theories. One interesting theory would be a deformation of $\mathcal{N} = 4$ quiver theory of \hat{A}_3 -type [47]. This theory may have two distinct Chern-Simons levels, as in GT theory. It would be interesting if, by taking one of the levels to be infinity, two of the four $U(N)$ gauge fields decouple, leaving Gaiotto-Witten theory [48] coupled to a large number of fundamental matters. Comparison of the behavior of Wilson loops in the quiver theory, Gaiotto-Witten theory and its deformation by adding fundamental matters might provide an insight into the effects of a large number of fundamental matters, or a role of the Veneziano limit, in AdS/CFT correspondence.

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Appendices

A $\mathcal{N} = 3$ Chern-Simons-matter theory

To construct the Lagrangian of an $\mathcal{N} = 3$ Chern-Simons-matter theory, it is convenient to employ the $\mathcal{N} = 1$ superspace formalism in four dimensions accompanied by the dimensional reduction. For notations and conventions, we basically follow [49].

Let ψ denote ψ_α , and $\epsilon^{\alpha\beta}$ etc. will be written explicitly. The three-dimensional gamma matrices are

$$\gamma^0 := i\sigma^2, \quad \gamma^1 := \sigma^3, \quad \gamma^2 := -\sigma^1. \quad (\text{A.1})$$

The adjoint $\bar{\psi}$ is defined as usual:

$$\bar{\psi} := \psi^\dagger \gamma^0. \quad (\text{A.2})$$

In the dimensional reduction, we ignore all x^2 -dependence, and then redefine the indices suitably. For example, let v_m be a four-dimensional vector field. After the dimensional reduction, we denote v_2 by σ , and the other components v_0, v_1, v_3 by A_μ . The $\mathcal{N} = 2$ supersymmetry transformations for the vector multiplet fields are

$$\delta A_\mu = -i\bar{\lambda}\gamma_\mu\xi + i\bar{\xi}\gamma_\mu\lambda, \quad (\text{A.3})$$

$$\delta\sigma = \bar{\lambda}\xi - \bar{\xi}\lambda, \quad (\text{A.4})$$

$$\delta\lambda = -\frac{1}{2}\gamma^{\mu\nu}\xi F_{\mu\nu} - i\gamma^\mu\xi D_\mu\sigma + i\xi D, \quad (\text{A.5})$$

$$\delta D = D_\mu\bar{\lambda}\gamma^\mu\xi + \bar{\xi}\gamma^\mu D_\mu\lambda + [\sigma, \bar{\lambda}]\xi + \bar{\xi}[\sigma, \lambda], \quad (\text{A.6})$$

where $D_\mu\lambda = \partial_\mu\lambda - i[A_\mu, \lambda]$ etc. It is straightforward to check that the action

$$S_{\text{CS}} = \frac{k}{4\pi} \int d^3x \text{tr} \left[\epsilon^{\mu\nu\rho} \left(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho \right) - 2i\bar{\lambda}\lambda - 2D\sigma \right] \quad (\text{A.7})$$

is invariant under the above transformation. It is possible to write this action explicitly in terms of the vector superfield V [44].

To construct an $\mathcal{N} = 3$ Lagrangian, we introduce an adjoint chiral superfield Φ , and two chiral superfields Q and \tilde{Q} in R and \bar{R} representations, respectively, of the gauge group. This is nothing but the field content of an $\mathcal{N} = 2$ gauge theory with matters in four dimensions. The following Lagrangian

$$S_{\text{matter}} = \int d^3x \left[\int d^4\theta \left[Q^\dagger e^{-2V} Q + \tilde{Q} e^{2V} \tilde{Q}^\dagger \right] - \int d^2\theta \sqrt{2} \tilde{Q} \Phi Q - \int d^2\bar{\theta} \sqrt{2} Q^\dagger \Phi^\dagger \tilde{Q}^\dagger \right] \quad (\text{A.8})$$

in three dimensions possesses $\text{SU}(2)_R \times \text{SU}(2)_{\text{rot}}$ global symmetry if it is supplemented by ordinary kinetic terms of V and Φ . Here $\text{SU}(2)_R$ is the R -symmetry of the $\mathcal{N} = 2$ theory in four dimensions. On the other hand, $\text{SU}(2)_{\text{rot}}$ has a geometric origin if the action is regarded as a part of the dimensional reduction of an $\mathcal{N} = 1$ gauge theory with matters in six dimensions. Namely, it is the rotational symmetry in the three dimensional subspace which is reduced in the dimensional reduction.

Obviously, $\text{SU}(2)_R$ symmetry does not commute with $\mathcal{N} = 2$ supersymmetry in three dimensions. If there would exist an action which includes (A.7) and (A.8), and which is invariant under the $\text{SU}(2)_R$ symmetry, then the action should have a larger supersymmetry. However, the action $S_{\text{CS}} + S_{\text{matter}}$ does not have the $\text{SU}(2)_R$ symmetry. It is, for example, due to the constraint

$$\sigma = \frac{2\pi}{k} (qq^\dagger - \tilde{q}^\dagger \tilde{q}), \quad (\text{A.9})$$

where the indices for R and \bar{R} are not contracted in the right-hand side. Notice that the right-hand side is not a singlet of $\text{SU}(2)_R$. In fact, it is, say, the third component of a triplet of $\text{SU}(2)_R$.

This is not the end of the story. Since σ is also a component of a triplet of $\text{SU}(2)_{\text{rot}}$, it is possible that a diagonal $\text{SU}(2)_d$ subgroup of $\text{SU}(2)_R \times \text{SU}(2)_{\text{rot}}$ can be a symmetry of a theory. It can be shown that

$$S_{\mathcal{N}=3} = S_{\text{CS}} + S_{\text{matter}} - \int d^3x \left[\int d^2\theta \frac{k}{4\pi} \text{tr} \Phi^2 + \int d^2\bar{\theta} \frac{k}{4\pi} \text{tr} (\Phi^\dagger)^2 \right] \quad (\text{A.10})$$

has such a symmetry. Therefore, this theory has at least $\mathcal{N} = 3$ supersymmetry. It was shown in [48] that a special choice of the gauge group and the field content has to be made for a Chern-Simons-matter theory to possess $\mathcal{N} = 4$ supersymmetry. Therefore, generically, this theory has only $\mathcal{N} = 3$ supersymmetry. The $\mathcal{N} = 3$ supersymmetry transformation was explicitly written in [50].

Note that the action (A.10) is completely determined by the requirement of $\mathcal{N} = 3$ supersymmetry except for the value of k . Due to the invariance for large gauge transformations, k must be an integer. This implies that k is not renormalized beyond one-loop since the loop expansion parameter of the theory is k^{-1} . It is known that the presence of $\mathcal{N} = 2$ supersymmetry ensures the non-renormalization theorem of k [51]. Therefore, since (A.10) is classically superconformal after integrating Φ out, it is also superconformal quantum mechanically.

B ABJ slice

In subsection 3.2, we obtained the planar resolvent of GT theory

$$v(z) = \frac{\kappa_1}{2\pi} F\left(z, \frac{t}{\kappa_1}\right) + \frac{\kappa_2}{2\pi} G\left(z, \frac{t}{\kappa_2}\right), \quad (\text{B.1})$$

where

$$F(z, s) = \int_c^d dx \frac{\log(e^{-s}x)}{z-x} \frac{\sqrt{(z-a)(z-b)(z-c)(z-d)}}{\sqrt{|(x-a)(x-b)(x-c)(x-d)|}}, \quad (\text{B.2})$$

$$G(z, s) = - \int_a^b dx \frac{\log(-e^{-s}x)}{z-x} \frac{\sqrt{(z-a)(z-b)(z-c)(z-d)}}{\sqrt{|(x-a)(b-x)(c-x)(x-d)|}}. \quad (\text{B.3})$$

We assume $ab = 1$ and $cd = 1$ whose validity is shown in subsection 3.2. The functions $F(z, s), G(z, s)$ can be written as follows.

$$\frac{1}{\pi} F(z, s) = \oint_{C_+} \frac{dx}{2\pi i} \frac{\log(e^{-s}x)}{z-x} \frac{\sqrt{(z-a)(z-b)(z-c)(z-d)}}{\sqrt{(x-a)(x-b)(x-c)(x-d)}}, \quad (\text{B.4})$$

$$\frac{1}{\pi} G(z, s) = \oint_{C_-} \frac{dx}{2\pi i} \frac{\log(-e^{-s}x)}{z-x} \frac{\sqrt{(z-a)(z-b)(z-c)(z-d)}}{\sqrt{(x-a)(x-b)(x-c)(x-d)}}, \quad (\text{B.5})$$

where the contour C_+ encircles the cut $[c, d]$ and C_- encircles $[a, b]$.

To check the validity of the solution, consider the ABJ slice $\kappa_1 = \kappa_2 = 1$ and see whether the solution in [14] is reproduced. On this slice,

$$\begin{aligned} 2v(z) &= \oint_C \frac{dx}{2\pi i} \frac{\log(e^{-s}x)}{z-x} \frac{\sqrt{(z-a)(z-b)(z-c)(z-d)}}{\sqrt{(x-a)(x-b)(x-c)(x-d)}} \\ &\quad + \pi i \oint_{C_-} \frac{dx}{2\pi i} \frac{1}{z-x} \frac{\sqrt{(z-a)(z-b)(z-c)(z-d)}}{\sqrt{(x-a)(x-b)(x-c)(x-d)}}, \end{aligned} \quad (\text{B.6})$$

where $C = C_+ \cup C_-$ and $s = t/2$. Note that to define $G(z, s)$ without ambiguity, we should choose $\log(-e^{-s}x) = \log(e^{-s+\pi i}x)$, and the cut $[a, b]$ should be in fact $[a - i0, b - i0]$. The branch cut of the logarithm is chosen to be the negative real axis.

Consider the first contour integral. By deforming the contour,

$$\begin{aligned} & \oint_C \frac{dx}{2\pi i} \frac{\log(e^{-s}x)}{z-x} \frac{\sqrt{(z-a)(z-b)(z-c)(z-d)}}{\sqrt{(x-a)(x-b)(x-c)(x-d)}} \\ &= - \oint_z \frac{dx}{2\pi i} \frac{\log(e^{-s}x)}{z-x} \frac{\sqrt{(z-a)(z-b)(z-c)(z-d)}}{\sqrt{(x-a)(x-b)(x-c)(x-d)}} \\ & \quad - \oint_{C_{\log}} \frac{dx}{2\pi i} \frac{\log(e^{-s}x)}{z-x} \frac{\sqrt{(z-a)(z-b)(z-c)(z-d)}}{\sqrt{(x-a)(x-b)(x-c)(x-d)}}, \end{aligned} \quad (\text{B.7})$$

where C_{\log} encircles the negative real axis. The integral around z provides $\log(e^{-s}z)$. The integral along C_{\log} can be written, using the ordinary trick,

$$\begin{aligned} & - \oint_{C_{\log}} \frac{dx}{2\pi i} \frac{\log(e^{-s}x)}{z-x} \frac{\sqrt{(z-a)(z-b)(z-c)(z-d)}}{\sqrt{(x-a)(x-b)(x-c)(x-d)}} \\ &= \int_{-\infty}^0 \frac{dx}{z-x} \frac{\sqrt{(z-a)(z-b)(z-c)(z-d)}}{\sqrt{(x-a)(x-b)(x-c)(x-d)}}. \end{aligned} \quad (\text{B.8})$$

It turns out that the second term in (B.6) cancels a part of the integral. Indeed,

$$\begin{aligned} & \int_a^b \frac{dx}{z-x} \frac{\sqrt{(z-a)(z-b)(z-c)(z-d)}}{\sqrt{(x-a)(x-b)(x-c)(x-d)}} \\ &= \int_{a+i0}^{b+i0} \frac{dx}{z-x} \frac{\sqrt{(z-a)(z-b)(z-c)(z-d)}}{\sqrt{(x-a)(x-b)(x-c)(x-d)}} \\ &= -\frac{1}{2} \oint_{C_-} \frac{dx}{z-x} \frac{\sqrt{(z-a)(z-b)(z-c)(z-d)}}{\sqrt{(x-a)(x-b)(x-c)(x-d)}}. \end{aligned} \quad (\text{B.9})$$

As a result,

$$\begin{aligned} 2v(z) &= \log(e^{-s}z) + \left(\int_{-\infty}^a + \int_b^0 \right) \frac{dx}{z-x} \frac{\sqrt{(z-a)(z-b)(z-c)(z-d)}}{\sqrt{(x-a)(x-b)(x-c)(x-d)}} \\ &= \log(e^{-s}z) - \frac{1}{z} \int_{-\infty}^a dx \frac{x^2-1}{x^2-(z+z^{-1})x+1} \frac{\sqrt{(z-a)(z-b)(z-c)(z-d)}}{\sqrt{(x-a)(x-b)(x-c)(x-d)}}. \end{aligned} \quad (\text{B.10})$$

The integral in the second line can be performed explicitly by the use of the formula

$$\int dx \frac{x^2-1}{x^2-cx+1} \frac{1}{\sqrt{(x^2-ax+1)(x^2-bx+1)}} = 2 \int \frac{dt}{(c-a)t^2 - (c-b)} \quad (\text{B.11})$$

where $t = \sqrt{(x^2-bx+1)/(x^2-ax+1)}$. The integration in the right-hand side can be performed easily. Finally, we obtain

$$v(z) = \log \left[\frac{e^{-\frac{t}{2}}}{\sqrt{(c+d)-(a+b)}} \left(\sqrt{(z-a)(z-b)} - \sqrt{(x-c)(z-d)} \right) \right]. \quad (\text{B.12})$$

This resolvent is related to $\omega(z)$ in [14] as

$$\omega(z) = t + 2v(z). \quad (\text{B.13})$$

Using the relation [14]

$$\sqrt{(c+d)-(a+b)} = \sqrt{2\xi} = 2e^{\frac{t}{2}}, \quad (\text{B.14})$$

we reproduce Eq.(3.18) of [14].

C Consistency of GT theory

GT theory is reduced to ABJ theory by setting $k_1 = -k_2 = k$. It was argued in [23] that ABJ theory would be inconsistent as a quantum field theory if $|N_1 - N_2|$ is larger than the Chern-Simons level k . This can be explained as follows.

Recall that ABJ theory has a brane configuration whose worldvolume theory flows to ABJ theory in the IR limit. The brane configuration consists of an NS5-brane, a $(1, k)$ 5-brane, N D3-branes wrapping a compact direction, and l segments of D3-branes suspended between the NS5-brane and the $(1, k)$ 5-brane. The wrapped D3-branes can move freely, but the D3-brane segments cannot since they must be fixed at a position so as to minimize their energy due to the tension. In a point of the moduli space for which the wrapped D3-branes are far apart from the D3-brane segments, the low energy effective theory of the worldvolume degrees of freedom is the sum of the worldvolume theory on the wrapped D3-branes and the worldvolume theory on the D3-brane segments. The latter theory is $\mathcal{N} = 3$ pure Yang-Mills Chern-Simons theory with the gauge group $SU(l)$ and the level k . It was shown in [53][54][55] that the supersymmetry is broken in this theory if $l > k$.

One can consider a similar situation in ABJ theory directly. The separation of the wrapped D3-branes corresponds to introducing a vev of a scalar field, like [56]. The low energy effective theory around the non-trivial vev includes $\mathcal{N} = 3$ pure Chern-Simons theory which always has a supersymmetric vacuum because of the absence of quantum corrections. Therefore, there would exist a discrepancy in the pattern of supersymmetry breaking between the result from the brane picture and the field theory result. It was argued in [23] that this may suggest an inconsistency of ABJ theory for the case $l > k$.

Although the corresponding brane configuration for GT theory is not known, a similar analysis can be performed. For definiteness, we assume $N_1 > N_2$ and $k_1 > 0$. Consider GT theory expanded around the vacuum

$$\langle A_1 \rangle = \begin{bmatrix} v \cdot \mathbf{I}_{N_2} \\ 0 \end{bmatrix}, \quad \langle A_2 \rangle, \langle B_i \rangle = 0, \quad (\text{C.1})$$

where \mathbf{I}_{N_2} is $N_2 \times N_2$ identity matrix. We consider the large v limit.

The kinetic term of A_1 provides mass terms for gauge fields. Let $A_\mu^{(1)}$ be decomposed as

$$A_\mu^{(1)} = \begin{bmatrix} a_\mu & W_\mu^\dagger \\ W_\mu & C_\mu \end{bmatrix}, \quad (\text{C.2})$$

where a_μ are $N_2 \times N_2$ matrix-valued, and C_μ are $(N_1 - N_2) \times (N_1 - N_2)$ matrix-valued. Then, the

covariant derivative of A_1 becomes

$$D_\mu A_1 = -iv \begin{bmatrix} a_\mu - A_\mu^{(2)} \\ W_\mu \end{bmatrix} + \dots \quad (\text{C.3})$$

Since W_μ acquire a large mass in the large v limit, W_μ should be set to zero at low energy. Then, one can show that the Chern-Simons term for $A_\mu^{(1)}$ becomes a sum of Chern-Simons terms for a_μ and C_μ with a common level k_1 .

If all the matter fields coupled to C_μ become infinitely heavy in the large v limit, then C_μ would be gauge fields in $\mathcal{N} = 3$ pure Chern-Simons theory at low energy. According to [23], the appearance of this theory as a part of the low energy effective theory may suggest that GT theory might be inconsistent if $N_1 - N_2 > k_1$. To see whether this is the case, it is enough to check the masses of bi-fundamental fermions, because of the presence of supersymmetry. Let ψ_i be fermions in the superfield A_i and let $\tilde{\psi}_i$ be fermions in B_i . We denote them by

$$\psi_i = \begin{bmatrix} \chi_i \\ \eta_i \end{bmatrix}, \quad \tilde{\psi}_i = \begin{bmatrix} \tilde{\chi}_i & \tilde{\eta}_i \end{bmatrix}, \quad (\text{C.4})$$

where $\chi_i, \tilde{\chi}_i$ are $N_2 \times N_2$ matrix-valued. The gauge fields C_μ couple to η_i and $\tilde{\eta}_i$.

The superpotential in (3.1) does not provide mass terms for η_i nor $\tilde{\eta}_i$. The mass terms of those fermions come from the Yukawa couplings

$$\bar{\psi}_i \sigma^{(1)} \psi_i - \bar{\psi}_i \psi_i \sigma^{(2)} + \bar{\tilde{\psi}}_i \sigma^{(2)} \tilde{\psi}_i - \bar{\tilde{\psi}}_i \tilde{\psi}_i \sigma^{(1)}, \quad (\text{C.5})$$

where $\sigma^{(1)}, \sigma^{(2)}$ are determined by the component scalars A_i and B_i (see, for example, (3.8) for the case $k_1 = -k_2 = k$). These terms provide masses of order v^2 for all components of η_i and $\tilde{\eta}_i$. Therefore, they are integrated out at low energy.

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